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ION SHEATH EFFECTS NEAR ANTENNAS RADIATING WITHIN THE IONOSPHERE

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SUMMARY

A theoretical treatment of the electron displacement in the vicinity of a linear cylindrical antenna immersed in the ionosphere has been developed. This treatment explains the surprisingly thick ion sheaths observed experimentally when large RF voltages are applied to the antenna. The force that displaces the electrons is obtained from numerical solutions to the nonlinear differential equation describing their motion, and the results are consistent with the observations.

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INTRODUCTION

Early observations of changes in the impedance of an electrically short antenna in the ionosphere (References 1 and 2) led to the method of determining the dielectric constant – and hence the electron density – of the ionosphere from measurements of a linear antenna's capacity (References 3, 4, and 5).

It is well known that an ion sheath a few centimeters thick normally forms spontaneously around a body immersed in the ionosphere; and it has been shown (Reference 5) that, provided the RF voltage applied to the antenna is small (less than 2 volts at 7.75 Mc), the sheath thickness observed agrees approximately with that predicted by Jastrow and Pearse (Reference 6). However, when large RF voltages are applied to the antenna (e.g., 200 volts at 7.75 Mc), the ion sheath appears to be very much larger. The redistribution of the electrons under the influence of RF fields is investigated in the present paper, and a relatively simple analysis yields values of sheath diameter in good agreement with the experimental results.

ION SHEATH ARISING FROM VEHICLE POTENTIAL

The Jastrow and Pearse treatment of the sheath problem proceeds in two steps: First, the vehicle potential is computed from a knowledge of the electron temperature and density of the ionosphere. Second, this calculated potential is used to derive the sheath thickness. In practice, thermal- and photo-emission effects change the actual potential of the vehicle in a way difficult to predict, so that an actual measurement of the potential is necessary. Such measurements have indicated that the vehicle potential is generally in the range 0 to -1 volts with respect to the ambient medium (References 5 and 7). The size of the sheath that forms around a long cylindrical surface for different values of electron density and vehicle potential may then be found from the following simplified calculation.

Assume that the ion sheath is a sharply bounded cylindrical region around the antenna and that there are no electrons within this region. The transition region from sheath to plasma will actually

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be smoothed out because of the thermal motions of the electrons, but we will assume that there is an equivalent sharp boundary or sheath edge. This sheath edge is assumed to be at zero (plasma) potential.

The capacity of the antenna to this surface is

$$C = \frac{2\pi\epsilon_0}{\log\left(R/R_0\right)} \text{ farads/meter} \tag{1}$$

where

R = the sheath radius,

R_0 = antenna radius,

$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ farad/meter.

Thus the charge Q on the antenna is

$$Q = \frac{2\pi\epsilon_0 V}{\log\left(R/R_0\right)} \text{ coulombs/meter,}$$

where V is the antenna potential in volts; and the field E_1 at the sheath boundary is

$$E_1 = \frac{V}{R\log\left(R/R_0\right)} \text{ volts/meter.} \tag{2}$$

Inasmuch as the effect of the sheath is to shield the ambient medium from the effects of any charge on the antenna, we can also equate this field from the antenna with that arising from the electron density deficiency. The polarization field arising from the electron deficiency is

$$E_2 = \frac{\left(R^2 - R_0^2\right) Ne}{2R\epsilon_0} \text{ volts/meter,} \tag{3}$$

where N is the ambient density in electrons/cubic meter, and e is the electronic charge in coulombs. By equating Equations 2 and 3 to obtain the equilibrium condition, the value of R can be found from

$$2\epsilon_0 V = Ne\left(R^2 - R_0^2\right) \log\left(R/R_0\right) . \tag{4}$$

The solution to this equation is plotted in a convenient form in Figure 1, from which the ratio R/R_0 is readily obtained from different values of V/N at various values of R_0 (meters). The usual simplifying assumption of negligible magnetic field effects is made in the above treatment.

The sheath size can be increased if the antenna is biased negatively with respect to the vehicle. This has the effect of decreasing the influence of the ionosphere on the antenna reactance, and is helpful when an approximately constant antenna impedance is desired.

If the antenna voltage varies sufficiently slowly, the sheath thickness will also vary—the maximum frequency at which the sheath can follow the voltage changes being close to the electron plasma frequency (Reference 8). This does not mean that the electrons remain stationary when the applied frequency is greater than the plasma frequency but rather that they cannot rearrange themselves quickly enough to shield out the applied RF field. In the following treatment we are concerned with motions of the electrons at frequencies above the plasma frequency, so that it is their time-averaged positions that are affected by any sheath effects.

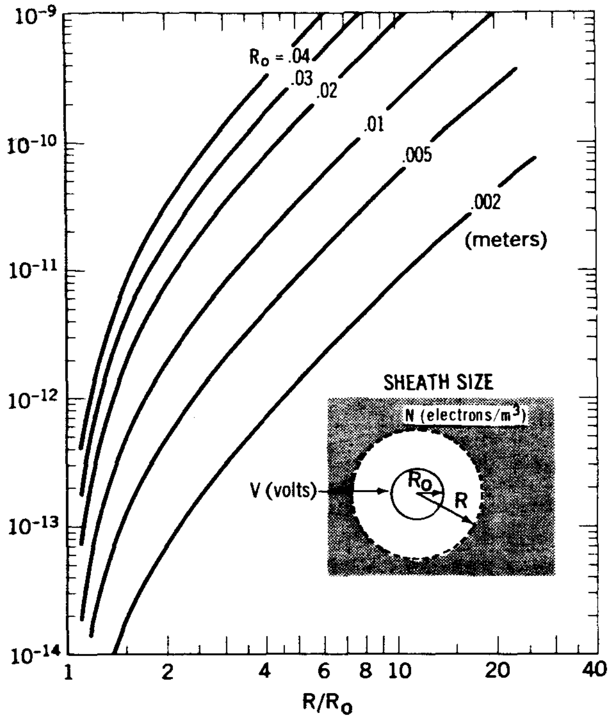

Figure 1—Effective sheath size as a function of antenna potential (in volts) and electron density (per cubic meter).

MOTION OF A SINGLE ELECTRON

At distances from a long cylindrical antenna that are small compared with the antenna length, the field is essentially perpendicular to the antenna axis. If effects arising from any magnetic field or from collisions or other losses are neglected, the electron motions will be along the field lines. The strength of the cylindrically radial field (i.e., perpendicular to the antenna axis) is inversely proportional to the distance from the axis. The motion of a single electron is thus described by the nonlinear differential equation

$$m\ddot{r} = \frac{eE_0}{r} \cos \omega t \ , \tag{5}$$

where

e = the electronic charge,

m = the electronic mass,

r = the distance of the electron from the antenna axis in meters,

E_0 = the peak field strength at a distance of 1 meter from the axis, and is found from

E_0 = $V_0 C/2\pi\epsilon_0$ volts/meter,

$V_0 \cos \omega t$ = the antenna voltage at any point measured with respect to the medium,

C = antenna capacity per unit length,

$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ farad/meter,

ω = operating frequency $\times\ 2\pi$.

Equation 5 may be solved by approximate methods or by numerical computation of electron trajectories.

Although the exciting voltage is sinusoidal, the fact that the electron experiences forces that are no longer sinusoidal (because of the inverse distance term) introduces an average force that accelerates an individual electron towards the lower field-strength region. This same phenomenon occurs when there are many electrons present, provided the frequency is above the plasma frequency, as is shown in the next section.

MOTION OF ELECTRONS IN AN IONIZED MEDIUM

When the antenna is immersed in an ionized medium, all the electrons will oscillate in the RF field and will also tend to move away from the antenna; but, since this motion produces a polarization of the medium, an equilibrium condition is attained. The situation then is that, with an RF field applied, a sheath appears—the sheath radius increasing with the field strength. The sheath edge, together with the electrons in the medium, vibrates at the applied frequency. To solve this problem, we require steady state solutions to a modified form of Equation 5, which should include the terms arising from the restoring forces. The deficit of negative charge in a cylindrical space of radius r free of electrons around the antenna is

$$Q = \pi r^2 Ne \text{ per meter,}$$

where it has been assumed that the space occupied by the antenna itself is negligible compared with the space free of electrons. The surface field at the sheath edge arising from this charge is

$$F = \frac{Q}{2\pi r \epsilon_0} = \frac{Ner}{2\epsilon_0} \text{ volts/meter,} \tag{6}$$

which is the same as Equation 3 with $R_0 = 0$.

The equation of motion for an electron at the sheath edge thus becomes

$$\begin{aligned} m\ddot{r} &= -Fe + \frac{eE_0}{r} \cos \omega t \\ &= -\frac{Ne^2 r}{2\epsilon_0} + \frac{eE_0}{r} \cos \omega t \ . \end{aligned} \tag{7}$$

Thus

$$\ddot{r} = -\frac{\omega_p^2 r}{2} + \frac{eE_0}{mr} \cos \omega t \,, \tag{8}$$

where ω_p = plasma frequency $\times\ 2\pi = \left(Ne^2/m\epsilon_0\right)^{1/2}$.

Equation 8 may be written in the following form, where the differentiations are with respect to a new variable $\tau (\tau = \omega t)$ and where the notation y represents $r(\tau)$ and $\ddot{y}$ represents $d^2 r/d\tau^2$:

$$\ddot{y} = -\frac{\omega_p^2 y}{2\omega^2} + \frac{eE_0}{m\omega^2 y} \cos \tau \,,$$

where τ is in radians. This leads to the simplified expression:

$$\ddot{y} = -Ay + \frac{B}{y} \cos \tau \;; \tag{9}$$

where

$A = X/2 \left(X = \omega_p^2/\omega^2\right)$,

$B = \dfrac{eE_0}{m\omega^2}$ = the oscillation amplitude of an isolated single electron in a uniform field of strength E_0.

Equation 9 may be regarded as specifying the trajectory of an electron under specified values of B (which depends on the antenna voltage and the frequency) and A (which depends on the electron density and frequency) for particular initial conditions. In general, the solution will describe an electron vibrating at about the applied frequency, with its center of motion oscillating at a frequency close to $\omega_p/2\sqrt{2\pi}$. This latter oscillation may, in some cases, carry the electron on to the antenna or completely out of the field. Our interest is in the steady state solution (i.e., the condition in which each cycle of the oscillation at the applied frequency is a repeat of previous cycles). To find these solutions, values of B and the initial radial distance y (when $\dot{y} = 0$) were chosen, and A was varied in a systematic manner until a repetitive (steady state) solution was obtained. The physical interpretation is as follows: The term Ay in Equation 9 may be written

$$Ay = A\bar{y} + A\delta y \,, \tag{10}$$

where $\bar{y}$ is the time-averaged value of y and δy is the instantaneous displacement of the electron from $\bar{y}$.

The term $A\delta y$ in Equation 10 then represents the effect of the polarization in returning the electron to its stable position at $\bar{y}$. Thus $A\bar{y}$ is a measure of the steady polarization required to balance

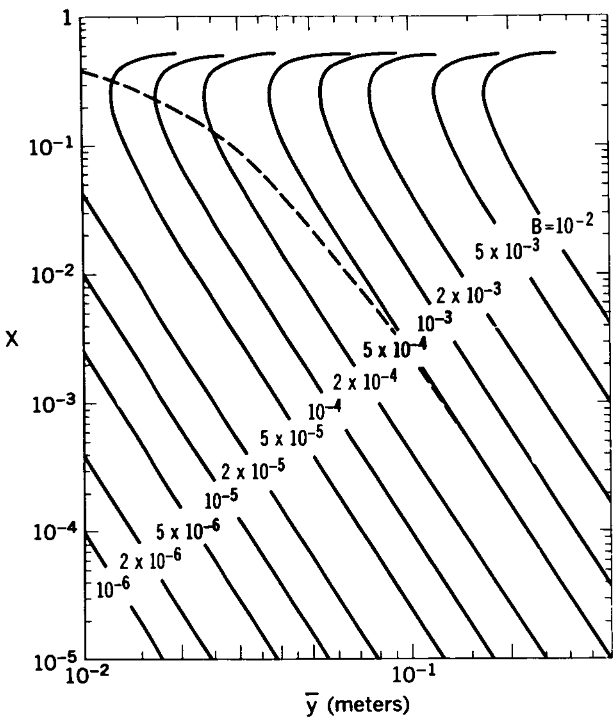

Figure 2—Effective sheath radius $\bar{y}$ as a function of X $(= \omega_p^2/\omega^2)$ for various values of the parameter B. (Dashed line shows the closest distance from the antenna for electrons vibrating about the mean position $\bar{y}$ for B = 5×10^{-4}.)

the mechanical force tending to move the electron away from the antenna. For a given B, the steady state trajectories for a series of initial y's may be found, each yielding a value for A and thus X (since $A = X/2$). The magnitude of $\bar{y}$ may be found from the calculated trajectory and thus a plot prepared in which $\bar{y}$ is shown as a function of X for various values of the parameter B. A series of such solutions is drawn in Figure 2.

The calculated value of $\bar{y}$ may be taken as the effective average sheath radius, since electrons closer to the antenna will experience—on the average—larger repelling forces and will tend to move out to this position. The more distant electrons in the medium will also oscillate in the RF field and will experience two forces tending to displace them from their mean positions. These two forces are:

1. The field that arises from the electron deficiency near the antenna and that varies as $1/r$, tending to move the electrons towards the antenna;

2. The average force that tends to move the electrons into the region of low RF field strength.

The resulting motion of an electron at $z (z > y)$ is then described by the differential equation

$$\ddot{z} = C - A\frac{y^2}{z} + \frac{B}{z}\cos\tau ,$$

where y satisfies

$$\ddot{y} = -Ay + \frac{B}{y}\cos\tau$$

and C is a constant that can be adjusted to obtain a stable solution. Solution of these equations yields small positive values of C and indicates that the electrons in the medium drift towards the antenna. This is to be expected, since the field from the electron-deficient region near the antenna decreases as $1/r$ while the force arising from the movement of the electrons decreases as $1/r^4$ approximately (as in the approximate analysis in the next section leading to Equation 15).

This effect may lead to some local increase in the electron density just outside the sheath region (i.e., a moderate electron sheath forms outside the positive ion sheath). In practice, it is probable that this boundary region, which has been assumed to be a step function for the purposes of the above analysis, will merge into a relatively smooth transition region.

APPROXIMATE SOLUTIONS

Approximate solutions to Equation 9 may be obtained by using certain simplifying assumptions. Assume that the motion of the electrons is sinusoidal, so that we may write

$$y = \bar{y} + a\cos\tau . \tag{11}$$

Then,

$$\ddot{y} = -a\cos\tau .$$

Substituting in Equation 9, neglecting the second harmonic terms that appear, and equating the steady and the oscillatory terms in the resulting equation, we obtain

$$\begin{aligned} a &= \frac{B}{(2A-1)}\,\bar{y} \\ &= -\frac{B}{(1-X)}\,\bar{y} \end{aligned} \tag{12}$$

and

$$\begin{aligned} a^2 &= \frac{2A\bar{y}^2}{1-A} \\ &= \frac{2X\bar{y}^2}{2-X} , \end{aligned} \tag{13}$$

Eliminating a, we find that

$$\begin{aligned} \bar{y}^4 &= \frac{(2-X)\,B^2}{2X\,(1-X)^2} \\ &\approx \frac{B^2}{X},\ \text{for small } X; \end{aligned} \tag{14}$$

and thus,

$$A = \frac{X}{2} \approx \frac{B^2}{2\bar{y}^4} . \tag{15}$$

Further harmonic terms could be added, assuming a series for y in Equation 11. This leads to a more accurate solution but becomes progessively more complicated, so that it is much simpler to use the computed curves than to attempt to increase the accuracy of these approximate solutions by including more harmonic terms.

DISCUSSION

It is seen from Equation 15 that, for small values of X, the mean distance $\bar{y}$ of the electrons from the antenna axis decreases as X increases, in agreement with the computed curves in Figure 2. With large X, $\bar{y}$ increases again. Some resonance effect would be expected from Equation 8 since, if the driving voltage E_0 is removed, the equation becomes

$$\ddot{r} = -\frac{\omega_p^2 r}{2}$$

$$= \text{constant} - \frac{\omega_p^2}{2} \delta r ,$$

where δr is the excursion of the sheath edge from its average position, with the oscillatory solution

$$\omega^2 = \frac{\omega_p^2}{2} .$$

Note that, for the particular geometry studied here, the simplified approach indicates that this resonance is not exactly at the plasma frequency but at

$$\omega = \frac{\omega_p}{\sqrt{2}} .$$

If the sheath is very small, it is not permissible to take $r_0 << r$, as was assumed in Equation 6. With $r_0 \approx r$, Equation 8 becomes (with $E_0 = 0$)

$$\ddot{r} = -\omega_p^2 \frac{r^2 - r_0^2}{2r}$$

$$\approx -\omega_p^2 (r - r_0)$$

$$= \text{constant} - \omega_p^2 \delta r .$$

In this case, the resonance occurs more nearly at the plasma frequency. The nonlinear motion of the electrons complicates the resonance phenomenon, and a more detailed investigation than is attempted in this particular paper is required.

In interpreting these results, remember that there generally is also a dc sheath formed around any antenna immersed in the ionosphere. This arises from the charge acquired by the vehicle from electron capture. The dc sheath thickness (i.e., in the absence of any RF fields applied to the antenna) is commonly of the order of 1 or 2 centimeters so that, although this dc effect may be negligible compared with the effects arising from the RF field at high voltages, it will be the controlling influence when the RF field is very small.

NUMERICAL EXAMPLE

Let us consider a typical situation, taking the following conditions:

Antenna radius (m)	0.01
Antenna capacity ($\mu\mu$f/m)	16
Peak RF voltage (v)	200
Frequency (Mc)	7.75

Then,

$$E_0 = \frac{V_0 C}{2\pi\epsilon_0}$$

$$= 57.6 \text{ volts/meter;}$$

so that

$$B = \frac{eE_0}{m\omega^2}$$

$$= \frac{E_0}{224.3f^2} \text{ meters (f in Mc)}$$

$$= 4.3 \times 10^{-3} \text{ meter.}$$

From Figure 2, as X increases, the mean distance of the inner layer of electrons from the antenna axis decreases from 0.22 meter at X = 0.01 to 0.125 meter at X = 0.1, with a minimum of 0.105 meter at about X = 0.3.

The minimum distance from the antenna axis attained by the nearest electrons to the antenna during their oscillations is also known. It is the initial value of y (for $\dot{y} = 0$) that was used in computing the solution of the differential equation.* This distance is shown, for example, as the dashed curve in

*The differential equation was solved by a Runge Kutta method on the Goddard Space Flight Center IBM 7090 computer.

Figure 2 for the conditions of $B = 5 \times 10^{-4}$. It is seen that, in this latter case, neglecting the effects of any dc bias on the antenna, electrons will strike the antenna (of radius 0.01 meter) when the electron density becomes greater than the value corresponding to $X = 0.4$.

EXPERIMENTAL OBSERVATIONS

The equation from which Figure 2 was derived is

$$\ddot{y} = -Ay + \frac{B}{y}\cos\tau .$$

If the scale of y is changed by a factor n, we may write

$$n\ddot{y} = -Any + \frac{n^2B}{ny}\cos\tau$$

or

$$\ddot{Y} = -AY + \frac{B'}{Y}\cos\tau ,$$

where $Y = ny$ and $B' = n^2B$.

It is thus possible to draw all the curves in Figure 2 as a single curve with suitable normalizing factors. This type of curve is shown in Figure 3, in which the computed results are normalized to $B = 10^{-2}$ with only the portion of the curve for $X > 0.1$ shown.

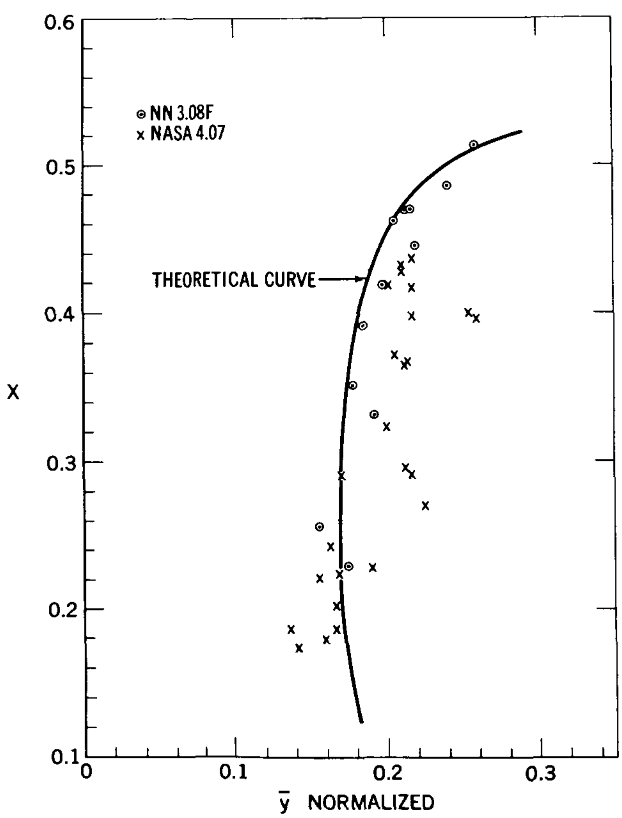

Figure 3—Experimental measurements of sheath radius normalized to $B = 10^{-2}$.

In Figure 3, a few experimental results normalized to $B = 10^{-2}$ are also plotted. These values were obtained during two rocket firings: NN 3.08F (Reference 3), and NASA 4.07 (Reference 5).

In the NN 3.08F experiment, the antenna impedance was obtained from measures of the voltages on the input and output sides of the antenna matching network. From these two measurements, both the resistive and reactive components of the antenna impedance can be derived (Reference 3). In the NASA 4.07 experiment, the approximate reactance was deduced only from a measurement at the output side of the antenna matching network—the matching circuit being fixed-tuned to resonate in free space so that the output voltage was a measure of the detuning. Since no allowance for any resistive changes could be made in the latter measurement, the results obtained cannot be considered as accurate as those obtained from NN 3.08F.

In both these flights the antennas were electrically short so that their impedances were primarily capacitive. The reactance measurements then yielded the apparent dielectric constant K' $(= 1 - X')$ of the medium from the relation

$$C' = K' C_0 \ ,$$

where C' is the measured capacitance and C_0 is the free space capacitance.

From this experimental value of X' and a knowledge of the true value (obtained from a two-frequency propagation experiment), the size of a cylindrical region free of electrons around the antenna, which would give the measured effective capacity, was calculated in the manner indicated below.

Since the antenna is long enough as compared with the sheath radius to be considered to be infinitely long, the effect of the sheath may be calculated by considering the sheath edge as one plate of a cylindrical capacitor around the antenna. Then it is found that

$$X' = \frac{C_s - C}{C_s - XC} X \ , \tag{16}$$

where C_s is the capacity of antenna to sheath per unit length and C is the free space capacity of antenna per unit length.

In the experimental measurements, the values of X and X' are obtained. The value of C can be deduced from free space measurements of the antenna impedance (in these cases, C was about $16 \mu\mu f/m$ for the NASA 4.07 rocket and about $19 \mu\mu f/m$ for the NN 3.08 rocket), so that C_s can be obtained from Equation 16. The sheath radius is calculated from the capacity of this cylindrical condenser by Equation 1. This radius was used (after normalizing to $B = 10^{-2}$) for the abscissa $\bar{y}$ in plotting the experimental results in Figure 3.

In the above treatment we have assumed that the sheath radius is constant along the length of the antenna, so that this approach is only applicable to electrically short antennas as used on NN 3.08F and NASA 4.07, in which the voltage is nearly constant along the length. Although the value of C (the free space capacitance of the antenna per unit length) is also variable since it depends on the spacing between the two halves of a dipole antenna, the values quoted for C may be regarded as effective average values.

For very long antennas, the sheath radius is greatest at the voltage antinodes; these points are also the places where capacitance changes have the most effect on the input impedance of the antenna. It has been found that applying Equation 16 to a long antenna gives results in good agreement with experimentally measured impedances.

Under the conditions of the NN 3.08 F rocket measurements, Table 1 is obtained for the ratio of the measured X' to the true X; this is compared to the ratio obtained from Equation 16 using the theoretical value of C_s (obtained from $\bar{y}$) and using $C = 19\,\mu\mu f/m$.

Table 1

Comparison of Measured X'/X and Calculated X'/X.

Height (km)	Apparent X'	True X	Measured X'/X	Calculated X'/X
120	0.067	0.229	0.29	0.31
130	0.089	0.256	0.35	0.32
140	0.104	0.331	0.31	0.34
150	0.124	0.351	0.35	0.35
160	0.140	0.391	0.36	0.35
170	0.147	0.418	0.35	0.36
180	0.147	0.445	0.33	0.35
190	0.155	0.452	0.34	0.35
200	0.155	0.459	0.34	0.35
210	0.155	0.459	0.34	0.35
220	0.162	0.486	0.33	0.33
230	0.166	0.513	0.32	0.25

It is of interest that the theoretical value of the sheath radius for NN 3.08 F (corresponding to $B = 5 \times 10^{-3}$) is about 13 cm, in good agreement with the value of about 6 inches (15.2 cm) published by Jackson and Kane (Reference 3). The column in Table 1 labeled *Measured* X'/X can also be interpreted as N'/N, where N' is the apparent electron density and N is the true electron density. This ratio (or discrepancy) could not be explained previously, since the sheath thickness required appeared to be unreasonably large.

CONCLUSION

At frequencies above the plasma frequency, under conditions such that collisions and magnetic field effects may be neglected, it is possible to deduce—from the relatively simple analysis given above—the way in which an RF voltage applied to a cylindrical antenna affects the mean position of electrons close to the antenna. The results obtained are, within the limits of the accuracy of the measurements, in good agreement with observations. The theory accounts for the discrepancies observed between true and apparent electron densities when a large RF voltage is applied to an RF impedance probe.

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